Lattice Study of the Conformal Window

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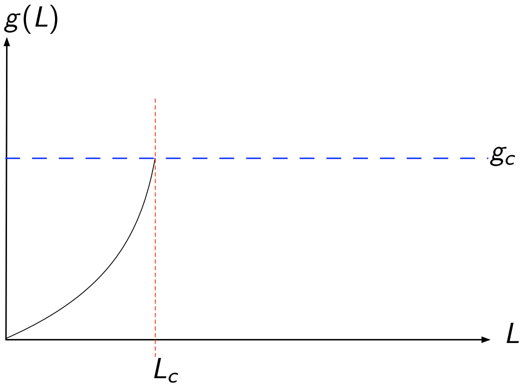
Outline

Motivation and Introduction

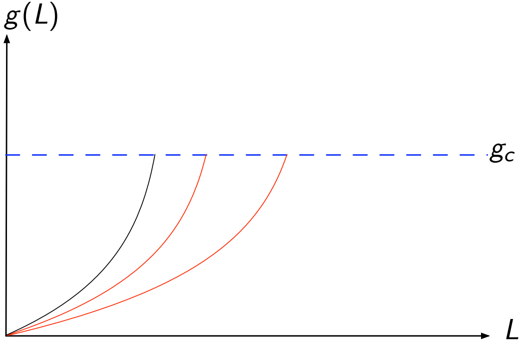
- New strong dynamics at the LHC? Technicolor, topcolor, composite Higgs...all involve SU(N) Yang-Mills sectors.
- With enough light quark flavors, Yang-Mills theory becomes IR conformal. Conformal, near-conformal behavior appears in many models (AdS/CFT, unparticles, walking TC...)
- We can get a lot out of studying Yang-Mills theory with many fermion flavors!
- Fix $N = 3$, N_f fermions, fundamental rep.
- "Lattice Study of the Conformal Window in QCD-like Theories" (Thomas Appelquist, George T. Fleming, EN.) PRL 100, 171607 (2008). Longer paper in preparation.

A cartoon of dynamical scales in SU(N) Yang-Mills



QCD is a one-scale theory where L_c is confinement scale.

A cartoon of dynamical scales in SU(N) Yang-Mills



Increasing N_f pushes confinement scale to longer distances.

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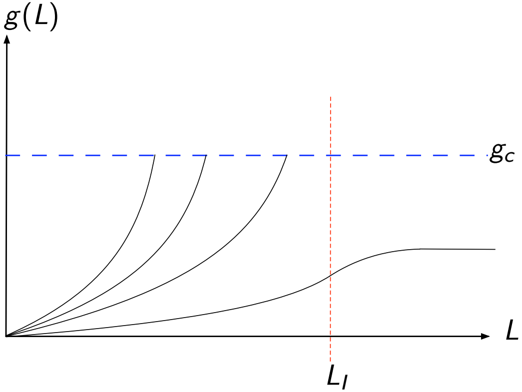
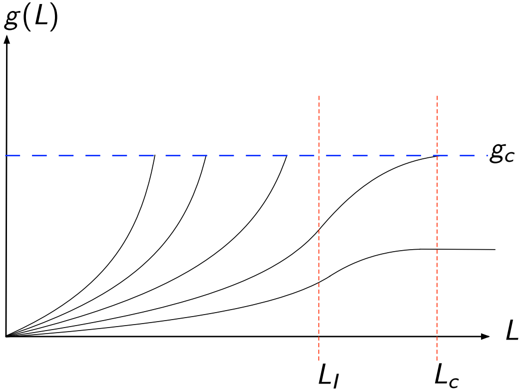


For large N_f an appropriate scale is the inflection point L_I.

A cartoon of dynamical scales in SU(N) Yang-Mills



A "walking" theory can have both scales L_I and L_c. Condensates are enhanced by modes between L_I and L_c[a].

[a] Appelquist, Terning, Wijewardhana, Phys. Rev. D **44**, 871 (1991)

Flavor dependence of $SU(3)$ Yang-Mills

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$0 < N_f < N_f^c$	Free ($g \to 0$)	Confined ($g \to \infty$)
$N_f^c < N_f < 16.5$	Free ($g \to 0$)	Fixed point ($g \to g^\star$)
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- The second row defines the conformal window.
- The value of N_f^c and the nature of the transition are important to model builders.
- N_f^c is unknown - pert. theory breaks down near the bottom of the window. Need non-perturbative study!

Estimates of N_f^c

- Continuum study based on counting degrees of freedom (Appelquist, Cohen, Schmaltz 1999) yields a bound:

$$N_f^c \leq 4N\left(1 - \frac{1}{18N^2} + \ldots\right)$$

- Gap equation studies (Appelquist et al, PRL 77:1214, 1996) suggest that this bound is saturated, i.e. for $N = 3$, $N_f^c \approx 12$.
- In supersymmetric SU(N) Yang-Mills, the ACS inequality yields $N_f^c \leq 3N/2$; Seiberg duality can be used to show the bound is saturated, $N_f^c = 3N/2$.
- However, previous lattice investigation of the conformal window (Iwasaki et al, PRD 69: 014507, 2004) claims the result $6 < N_f^c < 7$.

Program of study

- Goal: obtain an independent bound on N_f^c through lattice simulation.
- Method: measure the running coupling over a wide range of scales, and look for the existence of an IR fixed point.
- Use staggered fermions for computational efficiency, which naturally come in multiples of 4 flavors.
  - $N_f = 4$: clearly in the broken phase.[1]
  - $N_f = 8$: presence of IRFP unknown
  - $N_f = 12$: should be in the conformal window
  - $N_f = 16$: very perturbative IRFP (Heller[2] study)

[1] C. Sui, Ph.D thesis, Columbia 2001

[2] U. M. Heller, Nucl. Phys. (Proc. Suppl.) **63**, 248 (1998)

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Simulate here!

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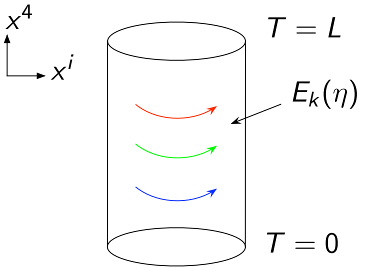
Measuring the running coupling

- In a typical lattice simulation, must work at scales well-separated from the lattice spacing a and the box size L. Hard enough, but we want to measure over a huge range of scales!
- To avoid box-size effects, we measure the Schrödinger Functional coupling $\overline{g}^2(L)$, defined directly at the scale L.
  - Lüscher et al, Nucl Phys B384 (1992)
  - S. Sint, Nucl Phys B421 (1994)
  - U. Heller, Nucl Phys B504 (1997)
  - Bode et al (ALPHA), Phys Lett B515 (2001)
- SF boundary conditions lift fermionic zero modes to scale $1/L$ - simulate with $m = 0$ directly!

The Schrödinger Functional



Schrödinger Functional (SF) simulations introduce Dirichlet boundaries in time. Boundary gauge fields are chosen to give a constant chromoelectric background field.

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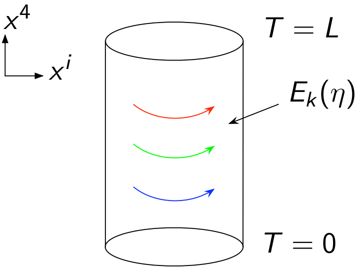

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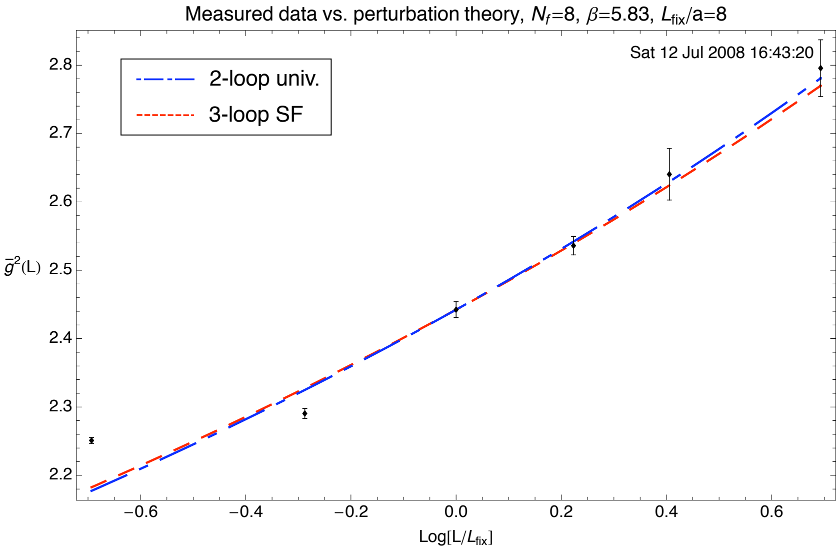
Running coupling

The SF running coupling $\overline{g}^2(L)$ is defined to vary inversely with the response of the action to the strength η of the background field,

$$\frac{dS}{d\eta} = \left.\frac{k}{\overline{g}^2(L)}\right|_{\eta=0}.$$

Data vs. perturbation theory



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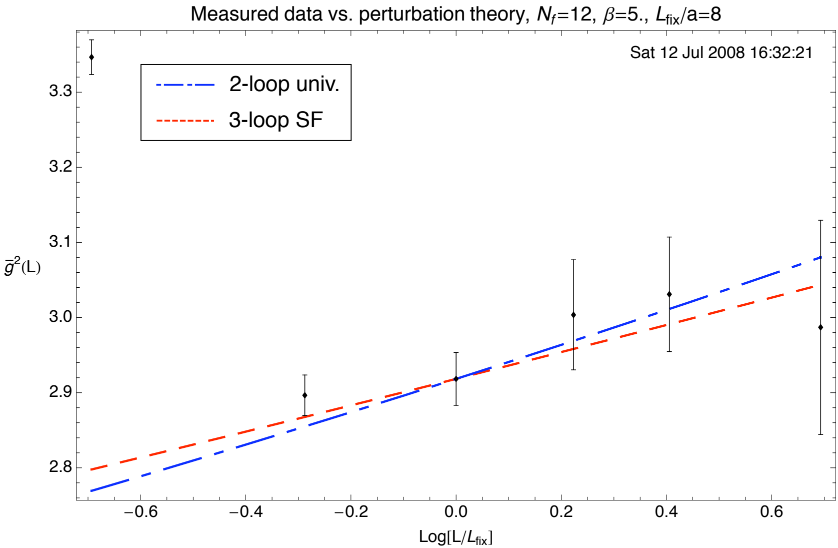


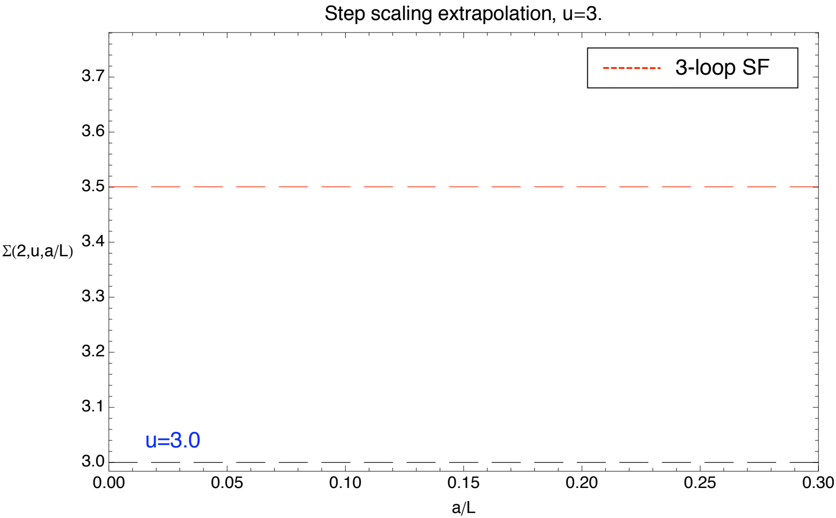
Measuring the running coupling, revisited

- Box size effects dealt with by using the SF, but fixing a and varying L still can't give a large enough evolution in scale.
- We use the step scaling procedure to link together results of simulations at many different a. Measure in discrete steps: $\overline{g}^2(L) \to \overline{g}^2(2L) \to \ldots$
- Define the step-scaling function,

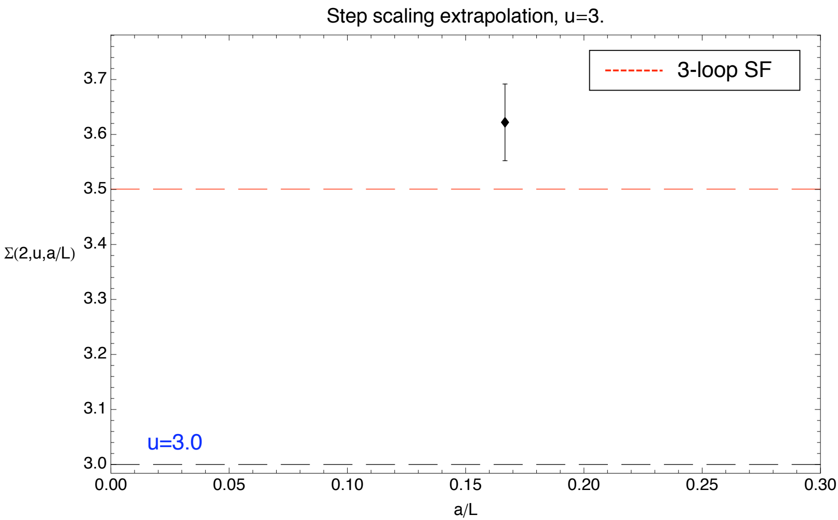
$$\Sigma(2, \overline{g}^2(L), a/L) \equiv \overline{g}^2(2L) + O(a/L)$$

The continuum limit $\sigma(2, u) \equiv \lim_{a\to 0} \Sigma(2, u, a/L)$ is basically a discretized version of the β-function.

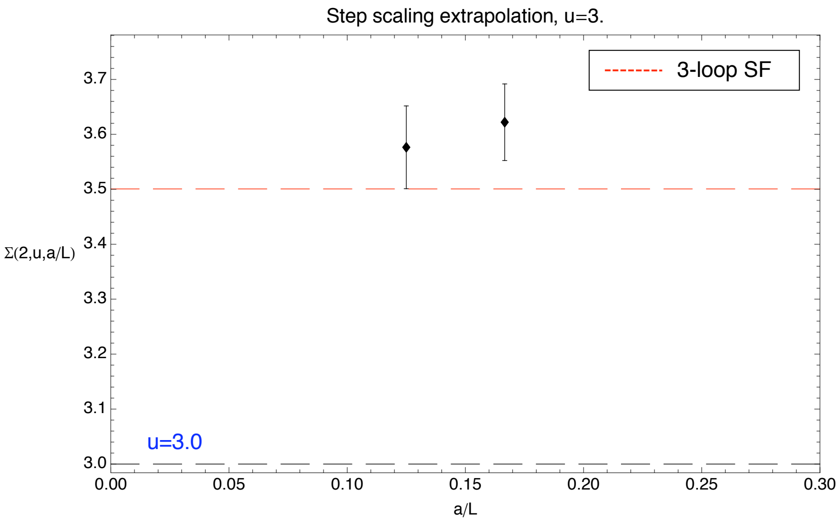
Continuum extrapolation



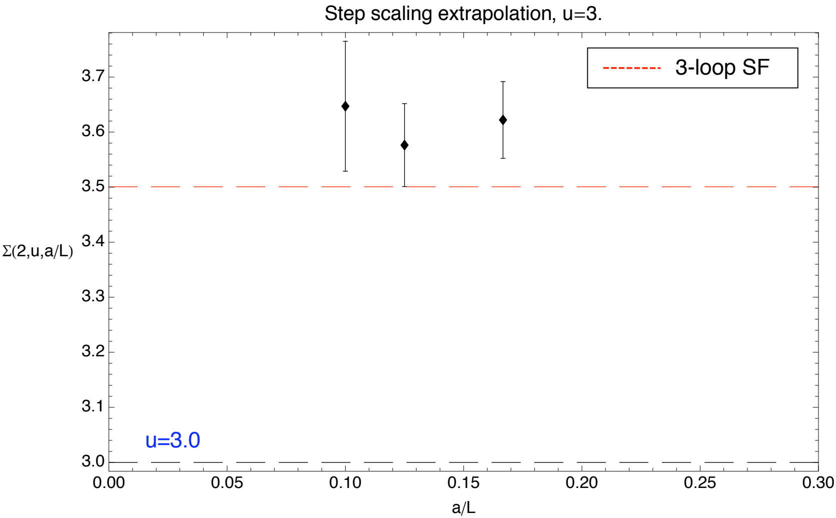
Continuum extrapolation



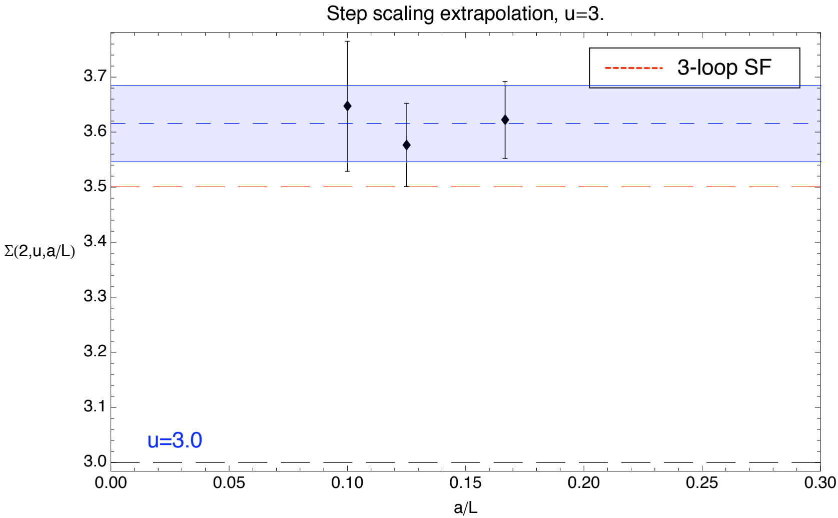
Continuum extrapolation



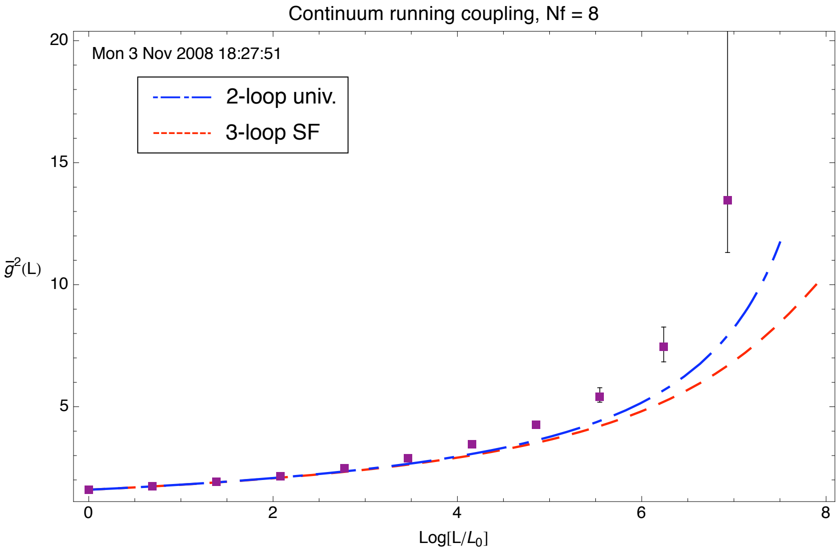
Continuum extrapolation



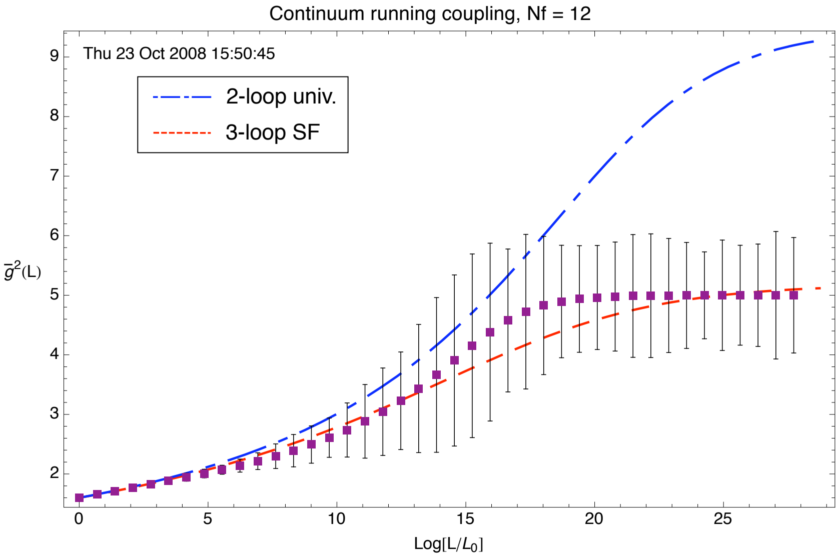
Continuum extrapolation



Results, $N_f = 8$

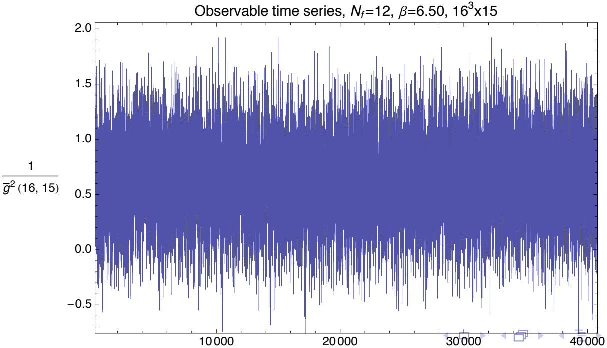

Continuum running coupling, Nf = 8
Mon 3 Nov 2008 18:27:51
2-loop univ.
3-loop SF
$\bar{g}^2$(L)
Log[L/L_0]
0
2
4
6
8
5
10
15
20

Results, $N_f = 12$



Aside: fun with data analysis

- The SF running coupling observable is extremely noisy - performing this measurement requires gathering and managing a lot of data.
- (7 spatial volumes L/a) $\times$ (2 temporal lengths T/a) $\times$ (≥ 3 numerical integrator step sizes) $\times$ (large range of couplings) $\times$ (lots of statistics in each simulation)...
- $\sim$100,000,000 distinct measurements at $N_f = 8$ alone!



Looking forward: $N_f = 10$

- The natural next step in constraining N_f^c is simulation at $N_f = 10$.
- Have to switch from staggered fermions to Wilson fermions (not a multiple of 4...)
- Aside from just narrowing down the conformal window...walking? IRFP strength?

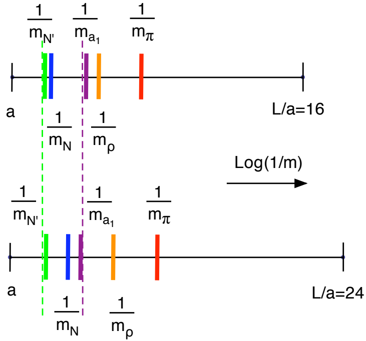
Wilson vs. staggered fermions

Wilson fermions are inherently more expensive than staggered, but we can offset this by making continuum extrapolation easier:

- Use clover-improved fermion action, boundary improvement counterterms (perturbative values)
- Simulate at odd L/a, more points in continuum extrapolation
- Use Chroma code package (with some modification)

Looking forward: multi-flavor spectrum

- In the end, the running coupling isn't enough - we'd like to know how the spectrum of these theories depends on N_f.
- Parity doubling? Enhancement of $\langle\overline{\psi}\psi\rangle$?
- Use chiral (domain-wall) fermions, to preserve chiral symmetry.
- Beware the unexpected, like finite-size effects:



Source: Cheng-Zhong Sui, Ph.D. thesis, Columbia (2001)

Looking forward: S-parameter

- Any model built using these theories has to yield a reasonable value for S...
- Original estimates were discouraging[3], but may be invalid for theories that differ significantly from QCD. Lattice measurement of S vs. N_f is essential.
- Actually, S has already been calculated on the lattice for $N_f = 2$! [4] They don't quote an explicit value for S, but give:

$$L_{10}(m_\rho) = -5.2^{+7}_{-5} \times 10^{-3}$$

which can be easily converted to $S = 0.33(3)$ with a reference $m_H = 380$ GeV. Compare with P&T's estimate: $S = 0.31(3)$.

[3] Peskin and Takeuchi, PRD 46 381, 1992

[4] Shintani et al, arXiv:0806.4222

Conclusions

Summary

- We have constrained the lower boundary of the $SU(3)$ fundamental conformal window: $8 < N_f^c < 12$, in agreement with the ACS bound ($N_f^c \leq 12$) and contradicting Iwasaki et al ($6 < N_f^c < 7$.)
- We have provided the first non-perturbative evidence of an IR fixed point outside of supersymmetric theories.

Future work

- Continued simulations at 8 and 12 flavors, to reduce error
- Study of running coupling at $N_f = 10$
- Spectrum calculations at various N_f
- Measurement of S-parameter
- Study of running coupling in other theories

Lattice Strong Dynamics (LSD) Collaboration

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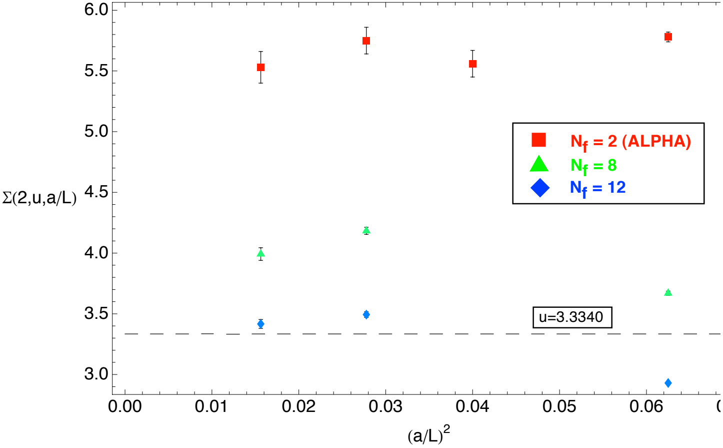
Data comparison with ALPHA



(Ref: Della Morte et. al. (ALPHA), hep-lat/0411025, NPB 713 (2005) p.378.)